

B.Sc. Semester - 6 (CBCS) Examination

April/May-2022 (OLD COURSE)

OPTIMIZATION AND NUMERICAL ANALYSIS -2(CORE)

Time: 2:30 Hours

Marks: 70

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 (A) Answer any one of the following questions.

[07]

- (1) Explain the steps of simplex algorithm.
- (2) Define: Slack and surplus variables and explain the Big-M method.

(B) Answer any one of the following questions.

[04]

- (1) Solve the following L.P.P. by two-phase method.

$$\text{Max. } Z = 3x_1 - x_2$$

$$\begin{aligned} \text{Subject to } & 2x_1 + x_2 \geq 2, \\ & x_1 + 3x_2 \leq 2, \\ & x_2 \leq 4, \\ & \text{and } x_1, x_2 \geq 0. \end{aligned}$$

- (2) Solve the following L.P.P. by graphical method.

$$\text{Max. } Z = 11x + 9y$$

$$\begin{aligned} \text{Subject to } & 3x + 2y \leq 8, \\ & 2x + 3y \leq 7 \\ & \text{and } x, y \geq 0. \end{aligned}$$

(C) Answer any one of the following questions.

[03]

- (1) Write general mathematical form of a linear programming problem.
- (2) Discuss how to assign coefficients to artificial variables in objective function in Big M method.

Q.2 (A) Answer any one of the following questions.

[07]

- (1) Explain Vogel's approximation Method (V.A.M.) to find initial solution of a transportation problem.
- (2) Find optimal basic feasible solution by MODI method.

	W_1	W_2	W_3	Supply
F_1	16	20	12	200
F_2	14	8	18	160
F_3	26	24	16	90
Demand	180	120	150	450

(B) Answer any one of the following questions.

[04]

- (1) Find dual of the following primal LPP.

$$\text{Min. } Z_x = x_1 + 2x_2$$

$$\text{Subject to } 2x_1 + 4x_2 \leq 160$$

$$x_1 - x_2 = 30$$

$$x_1 \geq 10$$

$$\text{and } x_1, x_2 \geq 0$$

(2) Solve the following assignment problem.

MEN

JOB

	1	2	3	4	5
A	1	3	2	3	6
B	2	4	3	1	5
C	5	6	3	4	6
D	3	1	4	2	2
E	1	5	6	5	4

(C) Answer any one of the following questions.

[03]

(1) Write full names of methods to find initial solution and optimum solution of a transportation problem.

(2) Write the mathematical formulation of transportation problem.

Q.3 (A) Answer any one of the following questions.

[07]

(1) State Gauss's forward and backward interpolation formula and hence deduce Sterling's formula.

(2) Prove that divided differences are symmetrical in all their arguments.

(B) Answer any one of the following questions.

[04]

(1) Given the values $f(14) = 68.7$, $f(17) = 64$, $f(31) = 44$ and $f(35) = 39.1$, find $f(27)$ using Lagrange's formula.

(2) Derive relation between divide differences and forward differences.

(C) Answer any one of the following questions.

[03]

(1) If $f(x) = x^3$, show that $f(a^3, b^3, c^3) = a + b + c$.

(2) Write Sterling's interpolation formula and explain when it is useful?

Q.4 (A) Answer any one of the following questions.

[07]

(1) In usual notation, prove that:

$$D^3 y_n = \left(\frac{d^3 y}{dx^3} \right)_{x=x_n} = \frac{1}{h^3} \left[\nabla^3 y_n + \frac{3}{2} \nabla^4 y_n + \frac{7}{4} \nabla^5 y_n + \dots \right]$$

(2) Derive general quadrature formula.

(B) Answer any one of the following questions.

[04]

(1) Evaluate $\int_4^{5.2} \log(x) dx$ by using Trapezoidal rule.

(2) Derive Simpson's $\frac{1}{3}$ rule.

(C) Answer any one of the following questions.

[03]

(1) Derive formula to find $\frac{dy}{dx}$ at $x = x_0$ from Newton's forward difference formula.

(2) Find the value of $\int_2^6 \frac{dx}{x}$ using Trapezoidal rule.

Q.5 (A) Answer any one of the following questions.

[07]

(1) Solve $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$ by Euler's modified method.

(2) Obtain predictor and corrector formula to solve differential equation

$$\frac{dy}{dx} = f(x, y), y(x_0) = y_0 \text{ by Milne's method.}$$

(B) Answer any one of the following questions.

[04]

(1) Solve: $y' = 1 + xy^2$ for $y(0.4)$ by Milne's method, given that $y(0) = 1$, $y(0.1) = 1.105$, $y(0.2) = 1.223$, $y(0.3) = 1.355$.

(2) Solve the differential equation $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$ by Runge-Kutta's method.

(C) Answer any one of the following questions.

[03]

(1) Let $\frac{dy}{dx} = x + y$, $y(0) = 1$. Find first approximation of y at $x = 0.1$ by Picard's method.

(2) Write working rule of Runge's method.
